PAVEMENT PRESERVATION PROGRAMMING: A MULTI-YEAR MULTI-CONSTRAINT OPTIMIZATION METHODOLOGY

by

Tonya Scheinberg  
Vice President, Ph.D.  
AgileAssets Inc. (PKA Texas Research and Development Inc.)  
3144 Bee Caves Road  
Austin, TX 78746  
Tel: (512) 327 4200  
Fax: (512) 328 7246  
E–mail: tscheinberg@agileassets.com

And

Panagiotis Ch. Anastasopoulos (Corresponding Author)  
Project Associate, M.Sc., Ph.D.  
AgileAssets Inc. (PKA Texas Research and Development Inc.)  
3144 Bee Caves Road  
Austin, TX 78746  
Tel: (512) 327 4200  
Fax: (512) 328 7246  
E–mail: panast@agileassets.com

Number of Words in Text: 6,186, Number of Tables: 4, Number of Figures: 1  
Total Equivalent Number of Words: 7,436

Prepared for presentation at the 89th Annual Meeting of the Transportation Research Board and publication in the Transportation Research Record.

(C) AgileAssets Inc.
ABSTRACT
A new formulation of a multi-year, multi-constraint, network-scale pavement management system methodology is described and compared with the year-by-year multi-constraint technique. This methodology uses mixed integer programming and predetermined strategies. The latter consist of a series of treatments according to decision trees. The goal is to identify the set of recommended strategies applied to a set of individual road sections, which minimize the overall strategy cost or maximize the condition-based benefit across the network, subject to some desired constraints. Three constraint types are presented, which can be applied simultaneously for all the analysis period and/or for specific years: the summation type, the weighted average condition, and the percentage above threshold. To demonstrate the methodology, a simple numerical paradigm is presented. A case study application to the State of Virginia illustrates the use of the methodology at network-level and allows a comparison of the model solutions with the year-by-year multi-constraint technique. Using AgileAssets Inc. Pavement Management System, the results show significant cost savings from the implementation of the methodology.

INTRODUCTION
Billions of dollars are spent annually by transportation agencies on infrastructure asset management to meet legislative, agency, and public expectations. A wide range of assets is typically managed, varying from equipment, material stocks, data, information and human resources, to the physical transportation infrastructure, such as roadways, pavements, structures and their associated features. Clearly, infrastructure asset management is associated with nearly every aspect of the transportation agency’s work (e.g., design, planning, finance, programming, engineering, etc.). However, its cornerstone is managing physical transportation infrastructure assets.

Pavement maintenance and rehabilitation is one of the most critical and costly forms of infrastructure asset management. Preserving the pavements in an appropriate manner, extends their service life, and most importantly improves motorists’ safety and satisfaction, and saves public tax dollars. This should, roughly, be the goal of an ideal pavement management program. However, pavement management programming (as a major component of infrastructure asset management) has become increasingly complex over the years due to significant traffic/population growth and limited resources. In view of this, in 1998, the American Association of State Highway and Transportation Officials (AASHTO) recognized the importance of infrastructure asset management for the transportation agencies, and adopted it as a priority strategic initiative. Since then (even many years before), past research has extensively addressed the issue of pavement management programming.

Typically, pavement condition and serviceability indexes, road functional classes, traffic levels, and agency and user costs are considered to minimize total treatment costs across the network. In practice, each road agency formulates its own analytical methods (e.g., condition and/or priority indexes, decision trees, linear and nonlinear mathematical optimization models, etc.), engineering decision criteria and weighing factors for priorities, based on its goals, the network condition and characteristics. Common prioritization approaches evaluate inter-project tradeoffs in selecting treatment strategies that are budget-constrained. For example, a non-computer based methodology that does not require exact measures of importance among different impacts was developed in the 1980s (1). In this method, projects are grouped and selected based on the ranks and budget constraints, after individual values for priority evaluation measures are plotted and given that their distribution is known. (2) identified and compared a number of ranking and other (e.g., optimization, heuristics, etc.) prioritization methods used in pavement management systems (PMS). Three priority-setting techniques (i.e., simple ranking based on current year condition data, modified ranking which considers the future condition of pavement sections, and near optimization which considers both time and space) were presented and compared by (3). The comparison shows that the optimization technique produces the best results.

Similar to priority ranking, mathematical optimization requires current condition data on pavement sections at the network level, treatment alternatives and their effect on pavement condition and associated costs. Approaches typically used to solve problems for optimal solutions include linear and
nonlinear programming, integer programming, and dynamic programming (3-9). In the late 1970s, an operating computer program was developed by (10). The program utilizes integer programming to determine optimal maintenance pavement strategies by maximizing the overall maintenance effectiveness of the network. In the 1980s, a multi-year optimization model (on a year-by-year basis) was developed by (11). The model uses weighed reductions in pavement distress as the measure of effectiveness, to determine the optimal resurfacing priorities in the Indiana’s PMS. More recently, (12) developed multi-year maintenance and rehabilitation optimization programming for pavement network management, where a cost-effectiveness-based integer programming on a year-by-year basis is used, and the objective is to select the most effective projects for each programming year.

In pavement performance prediction, the probabilistic approach considers uncertainty and is able to consider a large number of alternative treatments. A Markov process is a mathematical model for the random evolution of a memoryless system (13). In pavement performance prediction, this means that the probability that the section makes a transition to a particular condition state in a unit time following a particular action, depends only on the present condition state of the section and the action selected at that time. It does not depend on how the section reached that condition state. With the section continuing to make its annual transitions, various costs incur, which can be used to compute the total expected present work cost for a given strategy. Then, the strategy that minimizes the total expected cost subject to selected performance-related constraints is sought. In practice, the State highway agencies in Kansas and Arizona have incorporated Markov decision processes in their network level PMS, to produce prioritized work programs (14-15). Furthermore, Markov decision and discrete-time Markovian optimization models have also been utilized for pavement management programming (16-17).

Due to the complexity of the Markov probabilistic optimization and to its corresponding software computational limitations, computationally efficient heuristic approaches have been traditionally implemented more widely. Heuristic approaches produce near-optimal solutions, are appropriate for large-scale problems, and typically include incremental benefit-cost ratio and marginal cost-effectiveness methods or can be more advanced and base upon artificial intelligence techniques (18). A number of road agencies (e.g., North Carolina, Minnesota, Alberta in Canada, etc.) have used heuristic approaches to select treatment strategies (15, 19).

Finally, artificial intelligence includes techniques that are particularly appropriate for pavement management (due to uncertain and/or incomplete information) such as artificial neural networks, fuzzy logic, and evolutionary computing. It has been used for needs analysis as alternatives to the traditional priority ranking tools, such as decision trees (20), or for pavement maintenance and rehabilitation triggering for probabilistic life-cycle cost analysis (21). Another artificial intelligence technique in pavement management application at the network level is genetic algorithms, which is based on the mechanics of natural selection and evolutionary computing used in solving complex optimization problems (22). Genetic algorithms are a particular class of evolutionary algorithms, and have also been widely used in pavement management for network level programming (23-25).

Given the size of the pavement management budget and the importance of pavement preservation in the sustainability of transportation infrastructure, an improved comprehensive multi-year network-wide pavement preservation program could ultimately save millions of dollars by allowing for more efficient allocation of resources. This paper seeks to add to past research by developing a flexible multi-year multi-constraint optimization technique (MYOPT). A numerical paradigm presents the specifics of the methodology’s procedures. Furthermore, data from Virginia Department of Transportation (VDOT) are utilized to investigate several scenarios. The results show that considerable cost savings are achieved with the MYOPT compared to the year-by-year multi-constraint optimization methodology (MCOPT). The findings of the study are expected to enable transportation agencies to make better decisions regarding pavement preservation, allowing the selection of long lasting pavement treatment options that will cost the least given the time horizon of the analysis scope, initial pavement conditions, weather, traffic loads or any other factors.
METHODOLOGY

Review of Multi-Constraint Optimization (MCOPT)

The multi-constraint optimization’s objective is to create an optimal work program (e.g., a series of pavement treatments applied to a road network) using a single objective and multiple constraints (and/or constraint subdivisions) across one year of the analysis scope. The goal is to either minimize expenditure to achieve a desired set of performance targets on a road network, or to maximize a condition or benefit indicator given a fixed budget. Thus the desired output of the analysis is a series of treatments to be applied to individual assets in each year, which minimize the treatment cost or maximize the condition-based benefit (for each specific year) subject to some desired constraints. Since each road section (in any given year) may be either treated or not treated as a whole, the problem is by definition an integer programming problem. That is, for each combination of road sections $i$ and treatments $j$ (with $j = 0, 1, 2, \ldots, J$, and where $j=0$ is the “do nothing” treatment option), the variable $x_{ij} = 0$ indicates that treatment $j$ will not be applied to a specific road section $i$, and $x_{ij} = 1$ indicates that treatment $j$ will be applied to road section $i$. Therefore, the objective functions are mathematically formulated as follows,

minimize treatment cost: $\min \sum_{ij} c_{ij} x_{ij}$, or 

maximize weighted average condition (or any attribute): $\max \sum_{ij} W_{ij} a_{in}^j x_{ij}$, or 

maximize percentage above threshold: $\max \sum_{ij} W_{ij} x_{ij} (a_{in}^j > T_n)$, 

where, 

c_{ij} x_{ij}$ is the cost of applying treatment $j$ to road section $i$, 

$\sum_{ij} W_{ij}$ the sum of weights for all sections, 

$a_{in}^j$ the post-improvement condition, and 

$a_{in}^j > T_n$ a binary function that is 1 if post-improvement attribute value $a_{in}^j$ is above a certain threshold $T_n$ and 0 otherwise.

To keep the complexity of the problem manageable, the set of possible treatments for each road section can be restricted by utilizing decision trees (or other methods) to identify a reasonable subset of possible treatments. The integer programming formulation is then used to identify which treatments are best fit for specific road sections. Note that the treatments on a given road section are mutually exclusive, so that $\sum_{j=0}^{J} x_{ij} = 1$ for all $i$. That is, only one and only one treatment can be chosen for each road section.

And the cost of treatment $j$ applied to road section $i$, is given by the expression $c_{ij}$. Note that for $j = 0$ (i.e., do nothing), $c_0 = 0$.

There are three constraint types defined: (a) Type 1 – summation type (e.g., budget, user cost, treatment cost, benefit, etc.), (b) Type 2 – weighted average condition, and/or (c) Type 3 – percentage above threshold. These constraint types can be utilized individually or simultaneously.

For the first constraint type, consider a spending restriction across all or portions of a road network, such as budget constraints that may apply to specific treatment types or to subsets of the road sections. For example, consider a budget constraint that applies to pavement rehabilitation projects on urban collector roads. This is a constraint on the type of treatment (pavement rehabilitation) and the type of asset (urban collector roads), with any number of mutually exclusive budget categories be defined and applied to the problem. Note that, budget categories are mutually exclusive because they are dependent upon the treatment being assigned. Therefore, each treatment allows for only one budget category. Hence, if $R$ mutually exclusive groups of road sections and treatment types are defined, the sum of treatment costs (given by $c_{ij} x_{ij}$) for the treatments assigned to any one group $r$ (where, $r \in R$) must be less than or equal to some fixed monetary amount $C_r$. That is, for each group $r = 1, 2, \ldots, R$,
To compare the relative importance of the road sections, weights can be assigned. Weights indicate the relative importance of the road sections compared to all other elements in the road network. For example, for a single road section \( i \) the weight \( w_i \) may be defined by the number of lane-miles of pavement for that section, with higher weight values indicating higher importance in the analysis. As such, the second constraint type (i.e., the weighted average condition constraint) restricts the output weighted average condition of the resulting road sections list after treatments are applied. Similarly to the first constraint type, weighted average constraints allow for specifying a set of groups over which individual constraints may be determined. Thus, the road network is subdivided into \( R \) groups (with \( r = 1,2,\ldots, R \)). The weighted average condition goals are all specified with respect to a specific performance attribute \( n \) and may differ from one group to another, so that the attribute goal \( A \) for a performance attribute \( n \) is \( A_{nr} \). And the weighted sum of an attribute \( a \) must be greater than a given quality level \( A_{nr} \), given by,

\[
\sum_{(i,j) \in r} w_{ij} x_{ij} a_{in}^f \geq A_{nr}, \quad \forall r = 1,2,\ldots, R \quad \text{and} \quad \forall n = 1,2,\ldots, N ,
\]

where,

\[
\sum_{(i,j) \in r} w_{ij} x_{ij} a_{in}^f \quad \text{is the weighted condition sum of after treatment attributes of road sections in group} \; r, \quad \text{and} \quad \sum_{i \in r} w_i \quad \text{is the sum of weights for road sections in group} \; r .
\]

For example, consider a road network where the goal specified is that the weighted average Critical Condition Index or CCI (CCI is computed as the lesser of the load-related and non-load related distress indices, and is defined on a scale of 0 to 100, with 100 being best condition) on urban interstates should be greater than 85.

The third constraint type (i.e., percentage above threshold constraint) is similar to the second. However, the restriction is that for a certain attribute \( a_n \) (within every group \( r \)), the weighted percentage of road sections above a specific threshold value \( 0 \leq L_{nr} < 100 \), must be greater than or equal to a given percentage \( 0 \leq P_{nr} < 1 \) (given that the higher the value of the attribute, the better the pavement condition),

\[
\sum_{(i,j) \in r} w_{ij} x_{ij} (a_{in}^f > L_{nr}) \geq P_{nr}, \quad \forall \; r = 1,2,\ldots, R \quad \text{and} \quad \forall n = 1,2,\ldots, N ,
\]

where,

\[
(a_{in}^f > L_{nr}) \quad \text{is a binary function (1 if the post improvement attribute is above the threshold, and 0 otherwise), and} \quad \sum_{(i,j) \in r} w_{ij} x_{ij} (a_{in}^f > L_{nr}) \quad \text{is the sum of weights where the treated condition exceeds the performance threshold}.
\]

For example, provided that the weights are defined as lane-miles of pavement per road section, this type of constraint is applied by specifying that the CCI must be greater than 80 for 75% of the lane-miles in the network. This one year problem is solved using integer programming. In addition, multiple year solutions can be found by solving the problem in a year-by-year basis (i.e., one year at a time), so that the results of year \( t \) are the input for the problem in year \( t+1 \).
Multi-Year Multi-Constraint Optimization (MOPT) Framework

In multi-year multi-constraint optimization, the goal is to create an optimal work plan using a single objective and multiple constraints (and/or constraint subdivisions) across two or more years. However, for the multi-year analysis, the work plan is now defined as a set of strategies applied to the road network (i.e., a set of road sections). Strategy $k$ (among a set of possible strategies $K$, with $k \in K$) is defined as a series of treatments initiated in every year $t$ within the time period $T$ of the analysis scope (with $t \in T$). As such, the desired output of the multi-year multi-constraint optimization analysis is a set of strategies implemented in the network that minimize the overall strategy cost or maximize the condition-based benefit across the network and within all the years throughout the analysis time period, subject to some desired constraints. This work plan then is the set of recommended individual strategies applied to a set of individual road sections.

As in the multi-constraint optimization analysis, integer programming is used to identify which strategies for particular road sections best fit the problem objectives and constraints. In words, for each combination of road section $i$ and strategy $k$, the variable $x_{ik} = 0$ indicates that the strategy $k$ will not be applied to section $i$, and $x_{ik} = 1$ indicates that the strategy $k$ will be applied to section $i$. Note that, in the multi-year analysis, a strategy will always be selected (i.e., there always exists a defined strategy that directs application of absolutely no treatments across the network during the time period of the scope of the analysis, namely, the “do nothing” strategy). Therefore, the strategies on a given road section are mutually exclusive (i.e., one strategy must be selected for each road section over the time period $T$), such that $\sum_k x_{ik} = 1, \forall i$.

The objective functions in MOPT are similar to the objective functions in MCOPT. Equations 1-3 become, respectively,

- minimize strategy cost: $\min \sum_{i,k,t} c_{ik} x_{ik}$, (8)
- maximize weighted average condition (or any attribute): $\max \sum_{i,k,t} w_i (a_{ikn}^t) x_{ik}$, and (9)
- maximize percentage above threshold: $\max \sum_{i,k,t} w_i (a_{ikn}^t > L_{nr}) x_{ik}$, (10)

where,

- $c_{ik}^t$ is the cost of treatments assigned to road section $i$ by strategy $k$ in year $t$,
- $a_{ikn}^t$ is the value of attribute $n$ in year $t$ according to strategy $k$,
- $\sum_{i,k,t} c_{ik}^t$ is the total cost of implementing strategy $k$ to road section $i$,
- $w_i a_{ikn}^t x_{ik}$ is the weighted condition of treated section $i$ with strategy $k$ (with respect to performance attribute $n$), and
- $w_i (a_{ikn}^t > L_{nr}) x_{ik}$ is the weight (e.g., number of lane-miles) where the post-improvement attribute value $a_{ikn}^t$ (the treated condition) exceeds the performance threshold value $L_{nr}$.

The same three constraint types are also defined for the multi-year analysis: (a) Type 1 – summation type (e.g., budget, user cost, treatment cost, benefit, etc.), (b) Type 2 – weighted average condition, and/or (c) Type 3 – percentage above threshold. These constraint types can be utilized individually or simultaneously. However, the constraints in the multi-year analysis can be applied throughout the analysis period and/or to each (and/or any) year of analysis individually. As such, it is feasible to simultaneously have multi-year constraints (these constraints can be applied for all the analysis period and/or for specific years).
In regard to the first constraint type, if we define \( R \) mutually exclusive groups of strategies, the sum of the strategy costs, \( c_{ik}^r \), for the strategy assigned to any group \( r \) (with \( r \in R \)) must be less than or equal to some fixed amount \( C_r^t \). So, for each group \( r = 1,2,\ldots,R \), and year \( t = 1,2,\ldots,T \), we have,

\[
\sum_{(i,k) \in r} c_{ik}^r x_{ik} \leq C_r^t, \quad \forall \, r, t
\]  

(11)

As an example, consider that the budget of a transportation agency for a specific network for a 7-year period of analysis is 700 million dollars. Alternatively, the agency can decide that the budget for each year of the analysis cannot be higher than 70 million dollars. Another alternative can be that the agency decides that the overall budget throughout the analysis period is 500 million dollars, and that for the last year the budget cannot exceed 50 million dollars. The first alternative restricts the budget amount for the overall analysis period, whereas the second restricts it for each year individually. The third alternative is a combination of the previous two (budget constraint throughout the analysis scope, and budget constraint for a specific year).

The second constraint type (i.e., the weighted average condition constraint) needs to be specified if it is desired to constrain the output weighted average condition of the resulting road section list after strategies are applied. In accordance to the summation type constraints, the weighted average constraints allow for a set of groups specification over which individual constraints may be determined. As such, the network is subdivided into \( R \) (such that, \( r=1, 2, \ldots, R \)) groups. The weighted average condition goals are all specified with respect to a specific performance attribute \( n \), and may differ from one group to another so that the attribute goal \( A_n \) for performance attribute \( n \), in any year \( t \), is thus indexed by \( r \) and is denoted by \( A_{nr}^t \). The weighted sum of an attribute, \( a \), must be greater than a given quality level \( A_{nr}^t \), given by,

\[
\sum_{(i,k) \in r} w_i a_{ik}^r x_{ik} \geq A_{nr}^t, \quad \forall \, r = 1, \ldots, R, \ \forall \, n = 1, \ldots, N, \ \forall \, t = 1, \ldots, T,
\]  

(12)

where,

\[
\sum_{(i,k) \in r} w_i a_{ik}^r x_{ik}
\]

is the weighted condition sum of treated sections in year \( t \) with strategy \( k \) in group \( r \), and

\[
\sum_{iar} w_i
\]

is the sum of weights for all sections in group \( r \).

For example, consider a road network where a transportation agency can specify the goal that the average CCI (defined as previously) on road sections with AADT>10,000 vehicles per day should be greater than 85 for each year of the analysis, and on all other road sections it must be greater than 80. Alternatively, a different CCI target value can be set for each year (or for some years) of the analysis, and/or an overall target value throughout the analysis time period.

The third type constraint (i.e., percentage above threshold constraint) is similar to the weighted average condition constraint (Type 2). However, the restriction is that for a certain attribute \( a_{ik}^r \) (within every group \( r \)), the weighted percentage of assets above a certain threshold value \( 0 \leq L_{nr}^t \leq 100 \) (in the case that the attribute value is measured on a 0-100 scale) must be greater than or equal to a given percentage \( 0 \leq P_{nr} < 1 \), such that,

\[
\frac{\sum_{(i,k) \in r} w_i (a_{ik} > L_{nr}^t) x_{ik}}{\sum_{iar} w_i} \geq P_{nr}, \quad \forall \, r = 1, \ldots, R, \ \forall \, n = 1, \ldots, N, \ \forall \, t = 1, \ldots, T,
\]  

(13)
where,
\[ a^t_{ik} > L^*_{ir}, \] a binary function (1 if post-improvement attribute value is above the threshold value; 0 otherwise),
\[ \sum_{i \in r} w_i, \] the sum of weights for all sections in group \( r \), and
\[ \sum_{i \in r} w_i \left( a^t_{ik} > L^*_{ir} \right) x_{ik}, \] the sum of weights where the post-improvement attribute value (the treated condition) exceeds the performance threshold value.

Apparently, the last element is the full sum of weights that exceed the performance threshold. When divided by the complete sum of weights, it gives the percentage of weights that are above the desired specified threshold. Considering the previous example, given that the weights are defined as lane-miles of pavement per road section, the percentage above threshold constraint can be applied by specifying that the CCI must be greater than 85 for 70\% of the lane-miles in the network for each year of the analysis. Alternatively, different percentages and/or CCI target values can be set for each (or some) year(s) of the analysis, and/or an overall percentage and/or target value throughout the analysis time period.

The above MYOPT problem formulation is a general definition of the optimal work plan development problem in any infrastructure asset management system. An obvious barrier to solve this problem at a network level is the excessive number of decision variables that inevitably increases the computation (solver) time. This issue can be resolved by considerably reducing the number of strategies. In the general problem formulation, the number of strategies for each section is \( j^T \) and the total number of decision variables is \( l j^T \) (\( l \): number of road sections; \( j \): total number of possible treatments; \( T \): analysis period). Using decision trees, the number of strategies for each section is reduced to \( 2^T \), which drastically reduces the time needed to solve the problem and no loss of generality occurs. For specifics on this methodology, see (26).

NUMERICAL MYOPT PARADIGMS

To demonstrate MYOPT, a simple numerical paradigm is presented in the next sections. The paradigm was solved using LPSolve IDE v5.5 (27). Finally, an analysis is conducted using AgileAssets Inc. Pavement Management System to demonstrate MYOPT with a network-scale example and compare its results with MCOPT.

Data

Consider a simple road network consisting of 4 road sections, namely a, b, c, and d. Each road section has a defined weight as a relative measure of importance such as lane mileage. The weights range from 11 to 45 lane-miles adding up to 100 lane-miles. Each section may be treated with 4 specific strategies (other strategies can also be applied) given that the analysis scope is 3 years (the treatments defined in each strategy are determined by the decision tree presented in Figure 1):

- Strategy 1: the section can be treated in the first year and every year after that (treatment selection is decision tree dependent);
- Strategy 2: the section can be treated in the second year (“do nothing” in the first year) and every year after that (treatment selection is decision tree dependent);
- Strategy 3: the section can be treated only in the third (last) year – “do nothing” in the first and second year (treatment selection is decision tree dependent); and
- Strategy 4 – Do Nothing Strategy: the section is not treated during the time period of the analysis scope.

Two performance attributes for each road section are considered, the CCI (critical condition index defined as previously – A1 – that is, measured on a scale of 0-100, with 100 being the best condition and 0 the worst) and the IRI index (international roughness indicator index – A2 – measured on a scale of 0-100, with 100 being the best condition and 0 the worst).
Problem Formulation and Results
The goal is to minimize the overall strategy cost for all the road sections throughout the analysis period. The input data (i.e., treatment type, cost per lane-mile ($/lane-mi), CCI, and IRI) for the four sections and four strategies across the three years of the analysis period, are presented in Table 1.
# Table 1: MYOPT Scenario Input Data

<table>
<thead>
<tr>
<th>Section</th>
<th>Weight (lane-mi)</th>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CCI A1</td>
<td>IRI A2</td>
<td>Treatment</td>
<td>Cost ($/lane-mi)</td>
<td>CCI A1</td>
</tr>
<tr>
<td>1 a</td>
<td>45</td>
<td>90</td>
<td>80</td>
<td>D</td>
<td>80,000</td>
</tr>
<tr>
<td>2 b</td>
<td>30</td>
<td>68</td>
<td>58</td>
<td>E</td>
<td>190,000</td>
</tr>
<tr>
<td>3 c</td>
<td>11</td>
<td>58</td>
<td>65</td>
<td>G</td>
<td>200,000</td>
</tr>
<tr>
<td>4 d</td>
<td>14</td>
<td>92</td>
<td>57</td>
<td>B</td>
<td>120,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section</th>
<th>Weight (lane-mi)</th>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CCI A1</td>
<td>IRI A2</td>
<td>Treatment</td>
<td>Cost ($/lane-mi)</td>
<td>CCI A1</td>
</tr>
<tr>
<td>1 a</td>
<td>45</td>
<td>90</td>
<td>80</td>
<td>DoN</td>
<td>0</td>
</tr>
<tr>
<td>2 b</td>
<td>30</td>
<td>68</td>
<td>58</td>
<td>DoN</td>
<td>0</td>
</tr>
<tr>
<td>3 c</td>
<td>11</td>
<td>58</td>
<td>65</td>
<td>DoN</td>
<td>0</td>
</tr>
<tr>
<td>4 d</td>
<td>14</td>
<td>92</td>
<td>57</td>
<td>DoN</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section</th>
<th>Weight (lane-mi)</th>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CCI A1</td>
<td>IRI A2</td>
<td>Treatment</td>
<td>Cost ($/lane-mi)</td>
<td>CCI A1</td>
</tr>
<tr>
<td>1 a</td>
<td>45</td>
<td>90</td>
<td>80</td>
<td>DoN</td>
<td>0</td>
</tr>
<tr>
<td>2 b</td>
<td>30</td>
<td>68</td>
<td>58</td>
<td>DoN</td>
<td>0</td>
</tr>
<tr>
<td>3 c</td>
<td>11</td>
<td>58</td>
<td>65</td>
<td>DoN</td>
<td>0</td>
</tr>
<tr>
<td>4 d</td>
<td>14</td>
<td>92</td>
<td>57</td>
<td>DoN</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: DoN = “Do Nothing”; CCI = Critical Condition Index (scale 0-100); IRI = International Roughness Index (scale 0-100).
As such, the objective function is to minimize the treatment cost, such that,

\[
C = \sum_{i=1}^{3} \sum_{k=1}^{4} \sum_{l=1}^{4} c_{ik} x_{ik} .
\]  

(14)

Since for a given road section there may be only one strategy applied through the time period of analysis (i.e., for the 3 years of the analysis scope), the following constraint is added,

\[
\sum_{k=1}^{4} x_{ik} = 1, \forall i = 1, 2, 3, 4 .
\]  

(15)

By definition, the problem will always select Strategy 4 (the “Do Nothing” Strategy) for all road sections as this will produce the minimum cost (equal to zero). As more constraints are added to the problem, the cost will increase.

For example, consider a Type 2 constraint, where it is specified that the weighted average of performance attribute A1 (i.e., CCI) must be above 85. Thus, the following inequality must hold for a feasible solution,

\[
\sum_{i=1}^{4} \sum_{k=1}^{4} w_{ij} a_{ik} x_{ik} \geq 85 \cdot \sum_{i=1}^{4} w_{ij} .
\]  

(16)

By solving the problem, it is determined that the following combination of strategies will provide the least cost way to achieve an annual weighted average of 85 for the CCI (i.e., attribute A1):

- Strategy 2 for section a,
- Strategy 1 for sections b and c, and
- Strategy 4 for section d.

As shown in Table 2 (i), the total cost of these strategies is 14,630,000$, and the weighted average CCI is above 85 for all the 3 years of analysis.

To redefine the problem, another constraint is added on the network condition. The weighted percentage of sections with an IRI index greater than or equal to the threshold value of 80 must be greater than or equal to 70%, such that,

\[
\sum_{i=1}^{4} \sum_{k=1}^{4} w_{ij} (a_{ik} \geq 80) x_{ik} \geq 0.70 \cdot \sum_{i=1}^{4} w_{ij} .
\]  

(17)

Table 2 (ii) shows that 75% of the network has an IRI greater or equal to 70, hence this constraint is met. The optimal solution that meets this constraint, gives a total cost of 13,800,000$ and is as follows (Table 2 (ii) summarizes the results):

- Strategy 1 for sections a and b, and
- Strategy 4 for sections c and d.

Table 2 (i) also presents the computed percentage of weights with IRI index values greater than 80. Apparently, the solution presented in Table 2 (i) does not meet the Type 3 constraint (i.e., for year 1, only 41% of the sections will have IRI greater than or equal to 80).

The optimal solution that meets all (Type 2 – weighted average, and Type 3 – percentage above threshold) constraints, gives a total cost of 15,900,000$ and is as follows (Table 2 (iii) summarizes the results):

- Strategy 1 for sections a, b and d, and
- Strategy 4 for section c.

Table 2 (iv) summarizes the problem results for all the investigated scenarios.
### TABLE 2  Problem Results

(i) Results with weighted average constraint and with percentage of sections with IRI (A2) 80 and above

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight (lane-mi)</th>
<th>Year 0</th>
<th>Strategy Applied</th>
<th>CCI</th>
<th>IRI</th>
<th>Cost* ($)</th>
<th>Year 1</th>
<th>CCI</th>
<th>IRI</th>
<th>A2 Weight &gt;= 80</th>
<th>Year 2</th>
<th>CCI</th>
<th>IRI</th>
<th>A2 Weight &gt;= 80</th>
<th>Year 3</th>
<th>CCI</th>
<th>IRI</th>
<th>A2 Weight &gt;= 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>45</td>
<td>90</td>
<td>80</td>
<td>2</td>
<td>4,950,000</td>
<td>83</td>
<td>77</td>
<td>0</td>
<td>92</td>
<td>89</td>
<td>45</td>
<td>97</td>
<td>95</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>b</td>
<td>30</td>
<td>68</td>
<td>58</td>
<td>1</td>
<td>6,600,000</td>
<td>91</td>
<td>90</td>
<td>30</td>
<td>98</td>
<td>96</td>
<td>30</td>
<td>89</td>
<td>83</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>c</td>
<td>11</td>
<td>58</td>
<td>65</td>
<td>4</td>
<td>3,080,000</td>
<td>92</td>
<td>94</td>
<td>0</td>
<td>84</td>
<td>86</td>
<td>11</td>
<td>95</td>
<td>94</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>d</td>
<td>14</td>
<td>92</td>
<td>57</td>
<td>4</td>
<td>0</td>
<td>82</td>
<td>51</td>
<td>0</td>
<td>71</td>
<td>46</td>
<td>0</td>
<td>60</td>
<td>39</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Strategy Cost:</td>
<td>14,630,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighted Average / Percentage Above Threshold:</td>
<td>86.3</td>
<td>79.1</td>
<td>41%</td>
<td>90.0</td>
<td>84.8</td>
<td>86%</td>
<td>89.2</td>
<td>83.5</td>
<td>86%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Results with percentage above threshold constraint

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight (lane-mi)</th>
<th>Year 0</th>
<th>Strategy Applied</th>
<th>CCI</th>
<th>IRI</th>
<th>k</th>
<th>Cost* ($)</th>
<th>Year 1</th>
<th>CCI</th>
<th>IRI</th>
<th>A2 Weight &gt;= 80</th>
<th>Year 2</th>
<th>CCI</th>
<th>IRI</th>
<th>A2 Weight &gt;= 80</th>
<th>Year 3</th>
<th>CCI</th>
<th>IRI</th>
<th>A2 Weight &gt;= 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>45</td>
<td>90</td>
<td>80</td>
<td>1</td>
<td>7,200,000</td>
<td>95</td>
<td>92</td>
<td>45</td>
<td>89</td>
<td>83</td>
<td>45</td>
<td>94</td>
<td>91</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>b</td>
<td>30</td>
<td>68</td>
<td>58</td>
<td>1</td>
<td>6,600,000</td>
<td>91</td>
<td>90</td>
<td>30</td>
<td>98</td>
<td>96</td>
<td>30</td>
<td>89</td>
<td>83</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>c</td>
<td>11</td>
<td>58</td>
<td>65</td>
<td>4</td>
<td>0</td>
<td>50</td>
<td>53</td>
<td>0</td>
<td>44</td>
<td>41</td>
<td>0</td>
<td>35</td>
<td>32</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>d</td>
<td>14</td>
<td>92</td>
<td>57</td>
<td>4</td>
<td>0</td>
<td>82</td>
<td>51</td>
<td>0</td>
<td>71</td>
<td>46</td>
<td>0</td>
<td>60</td>
<td>39</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Strategy Cost:</td>
<td>13,800,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighted Average / Percentage Above Threshold:</td>
<td>87</td>
<td>81.37</td>
<td>75%</td>
<td>84.2</td>
<td>77.1</td>
<td>75%</td>
<td>81.3</td>
<td>74.83</td>
<td>75%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(iii) Results with weighted average and percentage above threshold constraints

<table>
<thead>
<tr>
<th>Sector</th>
<th>Weight (lane-mi)</th>
<th>Year 0</th>
<th>Strategy Applied</th>
<th>CCI</th>
<th>IRI</th>
<th>k</th>
<th>Cost* ($)</th>
<th>Year 1</th>
<th>CCI</th>
<th>IRI</th>
<th>A2 Weight &gt;= 80</th>
<th>Year 2</th>
<th>CCI</th>
<th>IRI</th>
<th>A2 Weight &gt;= 80</th>
<th>Year 3</th>
<th>CCI</th>
<th>IRI</th>
<th>A2 Weight &gt;= 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>45</td>
<td>90</td>
<td>80</td>
<td>1</td>
<td>7,200,000</td>
<td>95</td>
<td>92</td>
<td>45</td>
<td>89</td>
<td>83</td>
<td>45</td>
<td>94</td>
<td>91</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>b</td>
<td>30</td>
<td>68</td>
<td>58</td>
<td>1</td>
<td>6,600,000</td>
<td>91</td>
<td>90</td>
<td>30</td>
<td>98</td>
<td>96</td>
<td>30</td>
<td>89</td>
<td>83</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>c</td>
<td>11</td>
<td>58</td>
<td>65</td>
<td>4</td>
<td>0</td>
<td>50</td>
<td>53</td>
<td>0</td>
<td>44</td>
<td>41</td>
<td>0</td>
<td>35</td>
<td>32</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>d</td>
<td>14</td>
<td>92</td>
<td>57</td>
<td>1</td>
<td>2,100,000</td>
<td>97</td>
<td>84</td>
<td>14</td>
<td>99</td>
<td>95</td>
<td>14</td>
<td>85</td>
<td>79</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Strategy Cost:</td>
<td>15,900,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighted Average / Percentage Above Threshold:</td>
<td>89.1</td>
<td>85.99</td>
<td>89%</td>
<td>88.2</td>
<td>83.96</td>
<td>89%</td>
<td>84.8</td>
<td>80.43</td>
<td>89%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(iv) Scenarios comparison

<table>
<thead>
<tr>
<th>Sections</th>
<th>Strategies</th>
<th>Is Type 2 Constraint Met?</th>
<th>Is Type 3 Constraint Met?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution when Type 2 constraint is applied</td>
<td>a, b, c, d</td>
<td>2, 1, 1, 4</td>
<td>Year 1: Yes; Year 2: Yes; Year 3: Yes</td>
</tr>
<tr>
<td>Solution when Type 3 constraint is applied</td>
<td>a, b, c, d</td>
<td>1, 1, 4, 4</td>
<td>Year 1: Yes; Year 2: NO; Year 3: NO</td>
</tr>
<tr>
<td>Solution when Type 2 + 3 constraints are applied</td>
<td>a, b, c, d</td>
<td>1, 1, 4, 1</td>
<td>Year 1: Yes; Year 2: Yes; Year 3: Yes</td>
</tr>
</tbody>
</table>

Note: CCI = Critical Condition Index (scale 0-100); IRI = International Roughness Index (scale 0-100).

*These cost values represent the strategy cost for each specific road section.

(C) AgileAssets Inc.
Note that a solution to the problem may not be always feasible, as it depends on the constraints. For example, consider a constraint where the weighted percentage of sections with an IRI index greater than or equal to the threshold value of 90% must be greater than or equal to 90%, while the weighted average of the CCI must be greater than or equal to 90. This problem has no solution because there is no strategy combination across the network sections that can bring the pavement condition into such an excellent state.

**Large Scale Software Analysis – MYOPT and MCOPT Comparison**

Data from the Virginia Department of Transportation (VDOT) are utilized to demonstrate a large/network scale implementation of the developed MYOPT methodology and its benefits compared to the year-by-year MCOPT. Table 3 presents descriptive statistics of the sample data used for the network scale analysis (11,266 road sections are used for the analysis). For model estimation, the AgileAssets Inc. Pavement Manager™ system (see 26, 28-29) is used. This system utilizes an integer solver developed by the Computational Infrastructure for Operations Research, the COIN-OR Branch-and-Cut MIP Solver (30). In particular the branch and cut algorithm (a standard algorithm used to solve mixed integer programming problems of which this problem is a subset; see 30) is utilized for solving the integer problem formulated as described below.

**TABLE 3 Total Number of Lane-Miles of the Sample Network (Average Number of Lane-Miles per Road Section in Parenthesis)**

<table>
<thead>
<tr>
<th>Road Class / Pavement Type</th>
<th>BIT</th>
<th>BOC</th>
<th>BOJ</th>
<th>CRCP</th>
<th>JRCP</th>
<th>Grand Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interstate</td>
<td>3,127.2</td>
<td>457.8</td>
<td>859.1</td>
<td>403.9</td>
<td>406.1</td>
<td>5,254</td>
</tr>
<tr>
<td></td>
<td>(5.0)</td>
<td>(7.38)</td>
<td>(6.18)</td>
<td>(0.25)</td>
<td>(0.3)</td>
<td>(1.39)</td>
</tr>
<tr>
<td>Primary &amp; Secondary Roads</td>
<td>19,795.8</td>
<td>3.1</td>
<td>1,145.75</td>
<td>222.8</td>
<td>115.3</td>
<td>21,282.7</td>
</tr>
<tr>
<td></td>
<td>(3.73)</td>
<td>(1.04)</td>
<td>(2.53)</td>
<td>(0.19)</td>
<td>(0.19)</td>
<td>(2.83)</td>
</tr>
<tr>
<td>Grand Total</td>
<td>22,923</td>
<td>460.9</td>
<td>2,004.9</td>
<td>626.7</td>
<td>521.3</td>
<td>26,536.8</td>
</tr>
<tr>
<td></td>
<td>(3.86)</td>
<td>(7.09)</td>
<td>(3.39)</td>
<td>(0.22)</td>
<td>(0.26)</td>
<td>(2.35)</td>
</tr>
</tbody>
</table>

Note: BIT = Bituminous; BOC = Bituminous over continuously reinforced concrete; BOJ = Bituminous over jointed concrete; CRCP = Continuously reinforced concrete pavement; JRCP = Jointed reinforced concrete pavement.

The MYOPT and MCOPT analyses are conducted for several scenarios: for a prediction horizon of 2, 3, 4, 5, and 6 years, and for interstates, and primary and secondary roads. Therefore, 20 scenarios are investigated. Table 4 presents the estimated results using the two methodologies. The objective is to minimize the treatment cost throughout the analysis period for each scenario, subject to a Type 2 and a Type 3 constraint. Thus, for interstates, the weighted average of the CCI should be at least 70 for each year throughout the analysis period, and the percentage of the road sections with CCI value above 75 should be 50% or more of the network. Whereas, for primary and secondary roads, the weighted average of the CCI should be at least 65 for each year throughout the analysis period, and the percentage of the road sections with CCI value above 70 should be 50% or more of the network. Note that the threshold (trigger) values are different for the two road class groups due to differences in functionality (e.g., for safety reasons, interstates should be in a better condition state compared to local roads).

The treatments utilized by VDOT are corrective maintenance, preventive maintenance, restorative maintenance, and major rehabilitation (and of course, the “do nothing” option). Those are the treatments considered in the analyses.

Table 4 shows that, depending on the analysis time period and the road class group, the cost savings produced by the MYOPT compared to the MCOPT range from 7.7 to 53.4 percent; whereas, the average percentage cost savings across all the scenarios is 28.3 percent. These results are intuitive, as MYOPT seeks to comprehensively minimize the overall treatment costs throughout the analysis period, when MCOPT has this goal for each year separately. This finding is very important as it shows that MYOPT allows for greater cost savings for the agency.
TABLE 4  MCOPT and MYOPT Scenario Results

<table>
<thead>
<tr>
<th>Analysis Period</th>
<th>Road Class</th>
<th>MCOPT ($)</th>
<th>MYOPT ($)</th>
<th>MYOPT $ Savings</th>
<th>MYOPT % Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Years</td>
<td>Interstates</td>
<td>310,003,620</td>
<td>221,366,200</td>
<td>88,637,420</td>
<td>28.59%</td>
</tr>
<tr>
<td></td>
<td>Primary + Secondary</td>
<td>264,833,903</td>
<td>181,485,083</td>
<td>83,348,820</td>
<td>31.47%</td>
</tr>
<tr>
<td>5 Years</td>
<td>Interstates</td>
<td>252,650,801</td>
<td>164,649,791</td>
<td>88,001,010</td>
<td>34.83%</td>
</tr>
<tr>
<td></td>
<td>Primary + Secondary</td>
<td>178,184,674</td>
<td>126,955,857</td>
<td>51,228,817</td>
<td>28.75%</td>
</tr>
<tr>
<td>4 Years</td>
<td>Interstates</td>
<td>139,993,694</td>
<td>107,328,267</td>
<td>32,665,427</td>
<td>23.33%</td>
</tr>
<tr>
<td></td>
<td>Primary + Secondary</td>
<td>101,353,309</td>
<td>76,985,554</td>
<td>24,367,756</td>
<td>24.04%</td>
</tr>
<tr>
<td>3 Years</td>
<td>Interstates</td>
<td>81,666,294</td>
<td>63,449,763</td>
<td>18,216,530</td>
<td>22.31%</td>
</tr>
<tr>
<td></td>
<td>Primary + Secondary</td>
<td>41,869,848</td>
<td>19,513,543</td>
<td>22,356,306</td>
<td>53.39%</td>
</tr>
<tr>
<td>2 Years</td>
<td>Interstates</td>
<td>31,216,104</td>
<td>22,303,980</td>
<td>8,912,124</td>
<td>28.55%</td>
</tr>
<tr>
<td></td>
<td>Primary + Secondary</td>
<td>1,264,372</td>
<td>1,166,940</td>
<td>97,432</td>
<td>7.71%</td>
</tr>
</tbody>
</table>

SUMMARY AND CONCLUSIONS

A multi-year multi-constraint optimization methodology has been developed that relates optimal pavement maintenance and rehabilitation strategies to individual road sections in a road network. This methodology can be implemented at both project- and network-level analysis for any project- or network-level pavement preservation purposes. The goal is to create an optimal work plan using a single objective and multiple constraints over a time period of at least two years. Strategies are defined as a series of treatments according to a predetermined decision tree, initiated in some year within the time period of the analysis scope. Therefore, the desired output is a set of strategies implemented in the network that minimize the overall strategy cost or maximize the condition-based benefit across the network and within all the years throughout the analysis time period, subject to some desired constraints.

To identify which strategies for particular road sections best fit the problem objectives and constraints, integer programming is used. Three constraint types are considered. One that restricts cost or benefit attributes; one that restricts the average pavement condition weighted by some road/pavement attribute; and one that restricts the percentage of the network with a pavement condition above a desired threshold. These constraint types can also be utilized simultaneously. And, it is feasible to have multi-year constraints which can be applied for all the analysis period and/or for specific years.

The developed methodology assists in allocating available budgets based on desired pavement condition levels. It provides the ability to compare multiple analysis scenarios using different objective criteria, and the flexibility to develop cost-effective, policy-oriented work plans. With the help of flexible decision trees, proper treatment strategies are identified (in a timely fashion) for each pavement section with respect to its condition, climate, traffic levels, and agency policies. As such, pavement preservation needs can be easily estimated and available budgets can be distributed accordingly.
The simple numerical paradigm presented, offers a hands-on application of the developed methodology. Using the AgileAssets Inc. Pavement Manager™ system, an illustrative network-level application of the methodology to the road network in the State of Virginia shows the nature of the solutions produced, and enables the latter to be compared with solutions produced from other methodologies. The comparison shows that the multi-year multi-constraint optimization can produce up to 53% (28% in average) cost savings compared to the year-by-year multi-constraint optimization method. As such, this case study illustrates the potential value of the methodology and serves as the basis for developing further enhancements.

ACKNOWLEDGEMENTS

The authors are grateful to Eric Perrone, John Madison and Ronald Hudson (AgileAssets Inc.) for their helpful suggestions and comments. The contents of this paper reflect the views of the authors, who are responsible for the facts and the accuracy of the data presented herein, and do not necessarily reflect the official views or policies of the FHWA and the Virginia DOT, nor do the contents constitute a standard, specification, or regulation.

REFERENCES


